

PRECISION FLAVOR PHYSICS FROM THE LATTICE

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for the

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We present results for the light quark masses and the neutral kaon mixing parameter B_K from lattice QCD. Our data set includes lighter than physical light quark masses and 5 lattice spacings so that chiral extrapolation is not necessary and cutoff effects are fully under control. We obtain fully nonperturbative predictions for $m_{ud} = (m_u + m_d)/2$, m_s and B_K in the RI scheme with M_π , M_K and M_Ω as the only input quantities. Using perturbative 4-loop respectively 2-loop running and dispersive input from $\eta \rightarrow 3\pi$, we obtain $m_u^{\overline{\text{MS}}}(2 \text{ GeV}) = 2.17(4)(10) \text{ MeV}$, $m_d^{\overline{\text{MS}}}(2 \text{ GeV}) = 4.79(7)(12) \text{ MeV}$, $m_s^{\overline{\text{MS}}}(2 \text{ GeV}) = 95.5(1.1)(1.5) \text{ MeV}$ and $\hat{B}_K = 0.773(8)(8)$ where the first error is statistical and the second systematic.

1 Introduction

Lattice QCD is a tool to perform ab-initio calculations of QCD in the nonperturbative regime. One can stochastically perform the functional integral over gauge and fermion fields on a lattice regulated theory in finite volume. In order to obtain QCD predictions, it is then necessary to (a) remove the cutoff (b) extrapolate to infinite volume (c) tune the bare parameters of the theory or interpolate/extrapolate to these bare parameter values such that a predefined set of experimentally accessible, dimensionless quantities (e.g. hadron mass ratios) assume their physical value. If these requirements are met, one can use lattice QCD to obtain ab-initio QCD predictions of quantities not used as input in step (c) with a statistical error that arises from the stochastic integration.

Generally speaking, the challenge for lattice QCD is to simultaneously fulfill all the requirements (a)-(c) with control over systematic errors arising from each step and to minimize the total (statistical plus systematic) uncertainty on a target quantity with given computer resources. It has been shown recently, that relatively straightforward quantities such as the ground state light hadron spectrum can be reproduced with a few percent accuracy.¹ Here we present the results of a determination of the light and strange quark masses as well as for the neutral kaon mixing parameter B_K with controlled errors on the percent level. For the full technical details, we refer the reader to the original publications^{2,3,4}.

2 Quark masses

Light quark masses are fundamental parameters of the standard model Lagrangian that are inaccessible by direct experiment. In order to compute them, we compute some light hadron masses in lattice QCD with a number of bare input light quark masses. We then interpolate the bare

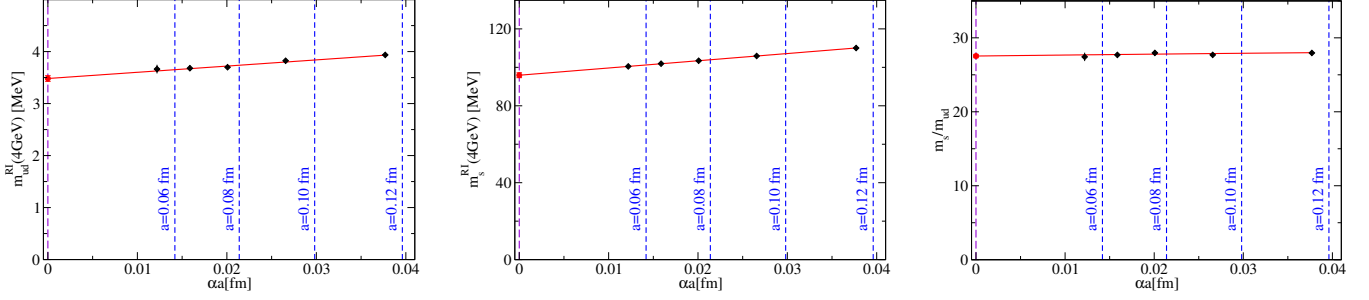


Figure 1: Continuum extrapolation of the average up/down quark mass, of the strange quark mass and of the ratio of the two.

quark masses at one value of the bare coupling g to the point where the measured light hadron masses take on their physical value and renormalize them. We compute the renormalization constant nonperturbatively in the RI-MOM scheme^{3,5}.

For technical reasons we work in the isospin limit and correct for the small isospin breaking at a later stage. We therefore take as input for finding the physical point the ratios of isospin averaged hadron masses M_π/M_Ω and M_K/M_Ω . The lattice cutoff or, equivalently, the lattice spacing a at the given bare coupling g itself is determined by comparing the dimensionless mass of the Ω baryon as measured on the lattice to the physical mass M_Ω . We interpolate to the physical point with various functional forms.³ The resulting spread enters (as a subdominant part) into our systematic error. Repeating this procedure at various different values of the bare coupling (in our case we use 5), we get the renormalized quark mass as a function of the lattice spacing a that we can extrapolate to $a = 0$ (see fig. 1).

For our lattice action, the leading term in the continuum extrapolation is formally of $O(\alpha_s a)$. There is however strong evidence that this term is numerically subdominant (for lattice spacings we work at) to the term of $O(a^2)$.⁶ We therefore use both forms in our analysis and include the resulting spread into our systematic error.

Finite volume corrections to stable particle masses are generically exponentially suppressed in $M_\pi L$ and can be corrected for systematically⁷ (see fig. 2). We included these corrections and found them generically to be at the permil level.

In order to obtain individual masses for the up and the down quark, we use the double ratio

$$Q^2 = \frac{m_s^2 - m_{ud}^2}{m_d^2 - m_u^2} \quad (1)$$

to convert our precise result for $m_s/m_{ud} = 27.53(20)(8)$ into an estimate of m_u/m_d . In principle, Q can be determined experimentally from $\eta \rightarrow 3\pi$ decays via dispersion relations. Due to the imperfect experimental data, there is some amount of modeling involved and we use a conservative estimate $Q = 22.3(8)$ from a recent review.⁸

As a last step, we convert the individual quark masses from the nonperturbative RI-MOM scheme into the $\overline{\text{MS}}$ scheme. In both schemes, the running of the quark mass is known to 4-loop order.^{9,10} As demonstrated in fig. 3, the nonperturbative running is well described by 4-loop perturbation theory above $\mu = 4$ GeV. We therefore compute our quark masses in the RI-MOM scheme at $\mu = 4$ GeV and further convert these numbers into the $\overline{\text{MS}}$ scheme using the results of Chetyrkin and Retey.¹⁰ Our final numbers are

$$\begin{aligned} m_u^{\overline{\text{MS}}}(2 \text{ GeV}) &= 2.17(4)(10) \text{ MeV} & m_s^{\overline{\text{MS}}}(2 \text{ GeV}) &= 95.5(1.1)(1.5) \text{ MeV} \\ m_d^{\overline{\text{MS}}}(2 \text{ GeV}) &= 4.79(7)(12) \text{ MeV} & m_{ud}^{\overline{\text{MS}}}(2 \text{ GeV}) &= 3.469(47)(48) \text{ MeV} \end{aligned} \quad (2)$$

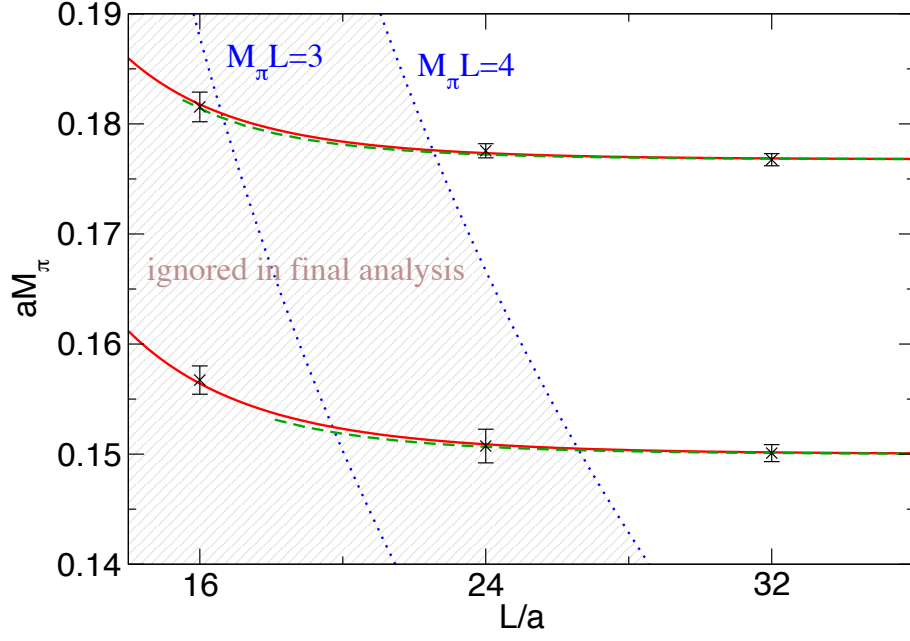


Figure 2: Finite volume effects in our data compared to analytic predictions. The dashed line corresponds to an approximate 3-loop prediction while the full line is an asymptotic approximation with one free coefficient.

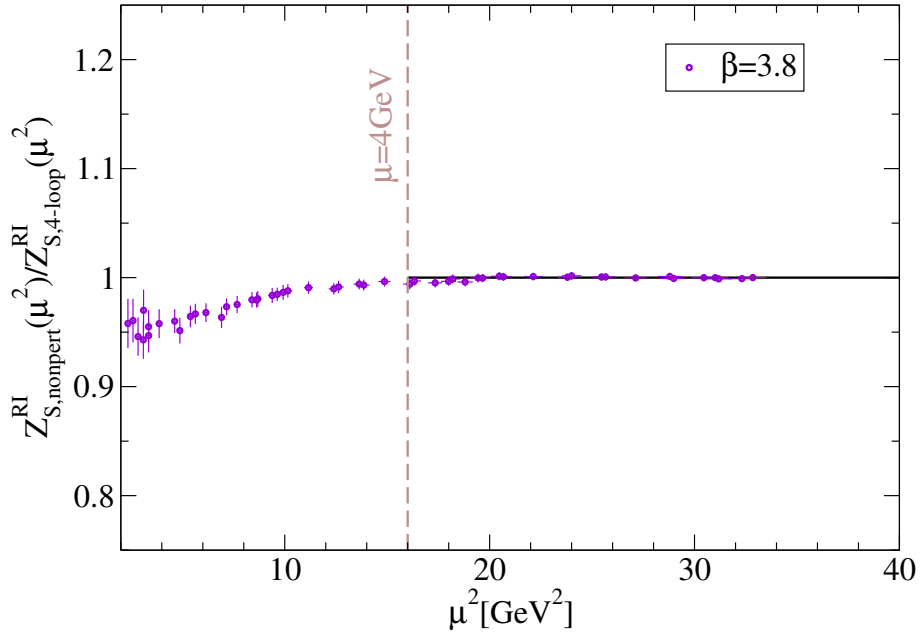


Figure 3: Nonperturbative over 4-loop perturbative running of the inverse of the mass renormalization constant Z_S in the RI-MOM scheme. Above 4 GeV agreement is reached within the statistical precision of our data.

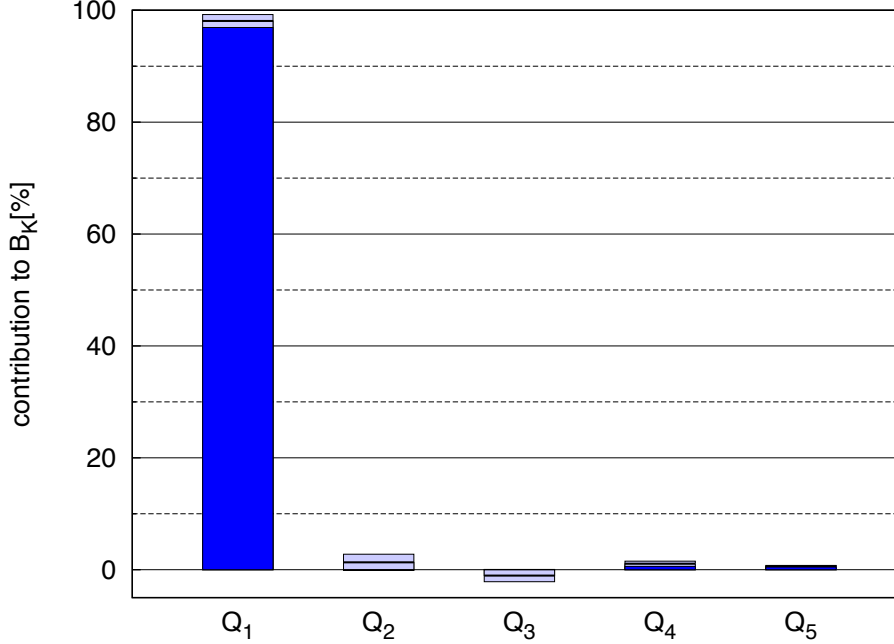


Figure 4: Contribution of the various operators towards the final value of B_K for the ensemble with the lightest pion mass. The contribution from the standard model operator O_1 is clearly dominant.

3 Neutral kaon mixing

Neutral kaon mixing is responsible for indirect CP violation in $K \rightarrow 2\pi$ decays. It is phenomenologically described by the parameter ϵ , which contains the hadronic matrix element of the standard model $\Delta S = 2$ operator $O^{\Delta S=2} = (\bar{s}\gamma_L^\mu d)(\bar{s}\gamma_{\mu L} d)$ that is usually parameterized as

$$B_K = \frac{\langle \bar{K}^0 | O^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} \langle \bar{K}^0 | A^\mu | 0 \rangle \langle 0 | A_\mu | K^0 \rangle} \quad (3)$$

A precise determination of B_K together with an experimental measurement of ϵ thus constitutes a precision test of the standard model in the kaon system which is particularly relevant for constraining various standard model extensions.^{11,12,13}

We have performed a lattice determination of B_K using the same setup as for our quark mass determination.⁴ One particular point to note is that our lattice discretized fermion action does only exhibit approximate chiral symmetry that gets fully restored in the continuum limit only. Consequently, mixing of the standard model operator, which has the structure $O_1 = (V-A)(V-A)$, with other dimension-6 operators that is forbidden in the continuum is allowed at finite lattice spacing. These other operators are $O_2 = VV - AA$, $O_{3/4} = SS \mp PP$ and $O_5 = TT$. As the standard model operator is chirally suppressed, these mixings can in principle be very large. Due to the good approximate chiral symmetry of our action,¹⁴ the mixing contributions to B_K are actually tiny as displayed in fig. 4.

We measure B_K on ensembles at 4 different lattice spacings and a variety of pion and kaon masses. Renormalization is again performed nonperturbatively in the RI-MOM scheme.¹⁵ For each lattice spacing, we interpolate the renormalized B_K to physical pion and kaon masses using various interpolators (see fig. 5) and the resulting physical value is extrapolated to the continuum (see fig. 6).^a In addition to the finite volume corrections on pion masses, we also apply finite volume corrections to B_K .¹⁶

^aIn fact, both the interpolation to the physical point and the continuum extrapolation are technically performed in one combined, global fit.

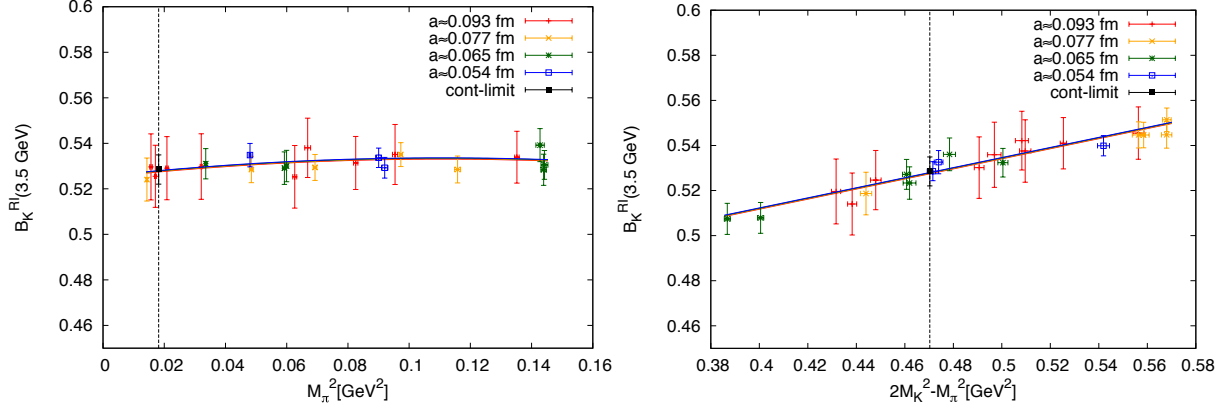


Figure 5: Interpolation of the renormalized lattice results for B_K to the physical pion and kaon masses. Note that the interpolation curves from different lattice spacings are almost on top of each other.

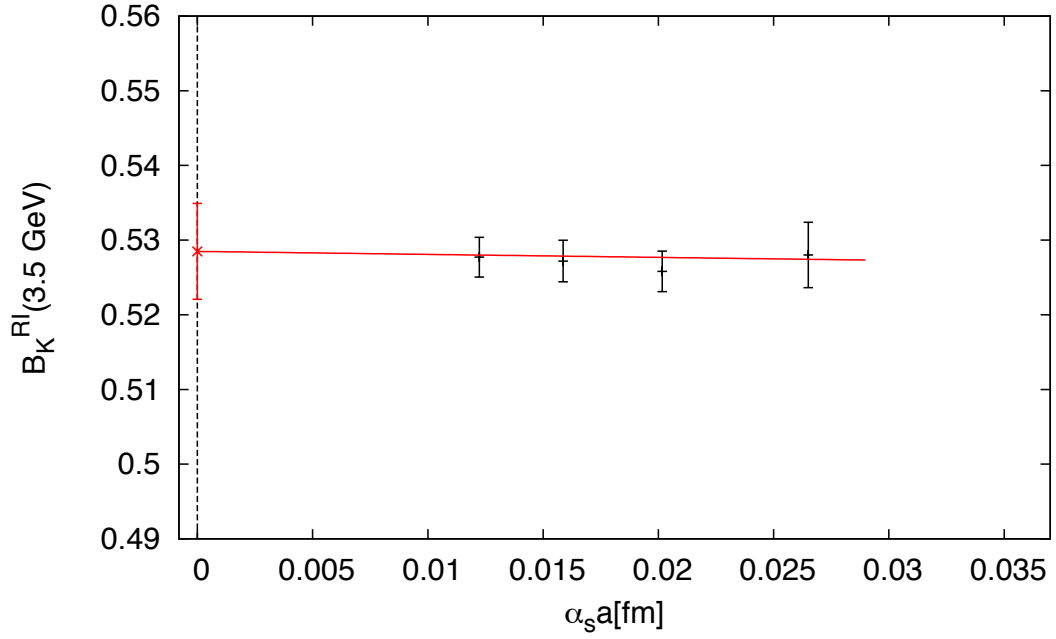


Figure 6: Continuum extrapolation of the renormalized lattice results for B_K at the physical pion and kaon masses.

Both interpolation to physical pion and kaon masses as well as the continuum extrapolation turn out to be very mild. In addition, the effect of finite volume corrections is even smaller than it was on quark masses. Consequently, the systematic error of our result is less than half the statistical error and we obtain

$$B_K^{\text{RI-MOM}}(3.5\text{GeV}) = 0.5308(56)(23) \quad (4)$$

as our final, fully nonperturbative result.

For further conversion of (4) into other schemes, we use results for the 2-loop running.^{17,18} Adding a conservative perturbative conversion uncertainty of 1%, we obtain

$$B_K^{\overline{\text{MS}}-\text{NDR}}(2\text{GeV}) = 0.5644(59)_{\text{stat}}(25)_{\text{sys}}(56)_{\text{PT}} \quad (5)$$

$$\hat{B}_K = 0.7727(81)_{\text{stat}}(34)_{\text{sys}}(77)_{\text{PT}} \quad (6)$$

The latter is compatible with the prediction $\hat{B}_K = 0.83^{+0.21}_{-0.15}$ from a global CKM fit.¹⁹

Acknowledgments

I would like to thank the organizers of the 47th “Recontres de Moriond” for creating a very pleasant and stimulating conference atmosphere. This work has been supported by the DFG grant SFB-TR 55.

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